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Recognition of individual heart rate patterns with cepstral vectors

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Abstract. Heart rate patterns may contain diagnostic as well as forensic information. To test these possibilities, individual heart rate patterns were represented as heart-rate cepstral vectors (HRCVs) computed in 12 dimensions via linear predictive coding (LPC) of brief segments of heart rate. A library of codebook vectors was computed for 12 cardiac patients from a standard ECG database. Statistical classification of subjects was based on the minimal weighted distances between test and codebook vectors. Weights were based on the ratio of inter- to intrasubject variances of their cepstral coefficients. Results showed that: (1) HRCV coefficients adequately reproduced the HRV spectrum, and (2) HRCV distances could be used to identify individuals within the group with a reliability of 93%. Thus, heart rate variations are an individual characteristic that can be represented as a single 12-dimensional vector.

1 Introduction

Each person's heart rate (HR) pattern reflects the temporal responses of his physiological controls to ongoing internal and external influences, operating on scales ranging from seconds to hours. Certain physiological states are characterized by periodicities in the HR, with the most obvious perhaps being respiratory sinus arrhythmia (RSA), oscillating at the breathing frequency (Saul and Cohen 1994). Other prominent periodicities in the HR have been associated with vasomotor control, with a cycle of 10–20 s (Craelius et al. 1986), and the diurnal cycle (Bigger et al. 1992). In addition to obvious rhythms, the HR pattern usually includes relatively wideband variations, occurring randomly as well as deterministically (Goldberger 1990). Indices of these heart rate variations have included power within frequency bands (Akselrod et al. 1981; Pomerantz et al. 1985), HR temporal variance within specified intervals (Kitney and Rompelman 1987), chaotic measures (Zbilut et al. 1988, Glass and Kaplan 1993), and entropy (Pincus and Goldberger 1994). While some of the indices are representative of cardiac disease, secondary to autonomic dysfunction or pharmacological blockade (Lombardi et al. 1987; Bekheit et al. 1990; Craelius et al. 1992), no single index has been found that describes the entire pattern of an individual's HR variability (HRV).

A new approach to HRV analysis is introduced herein, based on the heart rate cepstral vector (HRCV). Cepstrum represents the inverse Fourier transform of the logarithm of the power spectrum of the signal (Bogert et al. 1963; Noll 1967). Cepstral vectors can be derived from linear predictive coding (LPC), and thus represent the overall temporal characteristics of a signal. One prominent application of cepstral vectors has been in speaker recognition, whereby voice waveforms from a population are represented by a codebook of cepstral vectors (Atal 1976; Soong and Rosenberg 1988; Rabiner and Juang 1993). Individuals can be identified based on minimizing the distance between their subsequent vectors and the codebook vectors (Gray and Markel 1976). Similar strategies have been applied to recognizing patterns in knee vibrations (Tavathia et al. 1992) and EMG signals (Kang et al. 1995). In this paper, we hypothesized that specific HR patterns are characteristic of individuals, and can be described with cepstral vectors.

2 Calculations

2.1 Cepstral vector representation

Representation of signals by p coefficients, a_k, and residuals, x_n, is done recursively by LPC as follows (Oppenheim 1978; Press et al. 1992; Kang et al. 1995):

$$s_n = \sum_{k=1}^{p} a_k \cdot S_{(n-k)} + x_n \tag{1}$$

where
$s_n = n$th sample from the signal
$a_k = k$th predictor coefficient
$x_n =$ residual at nth sample
$p =$ number of LPC coefficients.

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Assuming an autoregressive model, the above equation can be computed using recursive time domain analysis or Burg's method (Press et al. 1992), without windowing. The set of LPC coefficients can then be converted to cepstral coefficients, comprising a cepstral vector, $\vec{\mathbf{c}}_m$, using the following relationships (Atal 1976; Rabiner and Juang 1993):

$$c_m = -\sum_{k=1}^{m-1} (1-(k/m)) \cdot a_k c_{m-k} - a_m \quad \text{for } 1 < m \leq p \quad (2)$$

where
a_k, a_m are LPC coefficients
$c_m = m$th cepstral coefficient of the set $\vec{\mathbf{c}}_m$
and $c_1 = -a_1$.

Thus, $\vec{\mathbf{c}}_m$, the vector defined by p cepstral coefficients. The cepstral envelope of the signal spectrum can be computed from the cepstral coefficients by McAulay and Quatieri (1992):

$$\log \bar{A}(\omega) = c_1 + 2\sum_{m=2}^{M} c_m \cdot \cos(m\omega) \quad (3)$$

where
$\bar{A}(\omega)$ is an estimate of the amplitude spectrum, $A(\omega)$
M = number of cepstral coefficients (same as p here).

The cepstrum in the time domain is the inverse Fourier transform of the log power spectrum. Harmonic components of signals are therefore recognized as a prominent peak in the cepstrum at a time delay corresponding to the fundamental period (Noll 1967).

2.2 Weighted cepstral distances

In order to compare two cepstral vectors, $\vec{\mathbf{c}}_m$ $\{m = 1 \ldots p\}$ and $\vec{\mathbf{c}}'_m$ $\{m = 1 \ldots p\}$, a weighted representation of the Euclidean distance between them can be calculated as (Soong and Rosenberg 1988):

$$d_{W_{cep}} = \sum_{m=1}^{p} w_m (c_m - c'_m)^2 \quad (4)$$

where

$w_m = m$th weighting coefficient of the set $\vec{\mathbf{w}}_m$ (see below).

Weighting the cepstral coefficients is a form of homomorphic filtering otherwise known as 'liftering' (Bogert et al. 1963). Weights are chosen statistically to optimize discrimination among individuals within a population. One liftering method is Mahalanobis weights, which have been used previously in speaker identification algorithms (Soong and Rosenberg 1988). They are calculated as the reciprocals of the average intrasubject variances of the cepstral vector coefficients.

A modification of the Mahalanobis method was developed for the HRCV as follows. First, a codebook (ensemble average) vector was computed for each individual over a time period consisting of S segments of HRV data:

$$\vec{\mathbf{c}}_{mj}^{\,\text{book}} = \frac{1}{S}\sum_{r=1}^{S} \vec{\mathbf{c}}_{mjr} \quad (5)$$

where
S = number of data segments comprising the subject's 'vector set'
m = coefficient index
j = subject index
r = segment index.

The population codebook vector is the average of the individual codebook vectors,

$$\vec{\mathbf{c}}_{mj}^{\,\text{book}} = \frac{1}{q}\sum_{j=1}^{q} \vec{\mathbf{c}}_{mj} \quad (6)$$

where q represents the number of individuals in the population.

The *inter*subject variance of the codebook vectors is thus:

$$\text{var}\langle \vec{\mathbf{c}}_{mj}^{\,\text{book}} \rangle = \sum_{j=1}^{q} (\vec{\mathbf{c}}_{mq}^{\,\text{book}} - \vec{\mathbf{c}}_{mj}^{\,\text{book}})^2 \quad (7)$$

and the *intra*subject variance is:

$$\{\text{var}\langle \vec{\mathbf{c}}_{mj} \rangle\} = \sum_{r=1}^{S} (\vec{\mathbf{c}}_{mj}^{\,\text{book}} - \vec{\mathbf{c}}_{mjr})^2 \quad (8)$$

and the average *intra*subject variance is:

$$\text{Avg}\{\text{var}\langle \vec{\mathbf{c}}_{mj} \rangle\} = \frac{1}{q}\sum_{j=1}^{q}\sum_{r=1}^{S} (\vec{\mathbf{c}}_{mq}^{\,\text{book}} - \vec{\mathbf{c}}_{mjr})^2 \quad (9)$$

The weights are then computed as:

$$\vec{\mathbf{w}} = \frac{\text{var}\langle \vec{\mathbf{c}}_{mj}^{\,\text{book}} \rangle}{\text{Avg}\{\text{var}\langle \vec{\mathbf{c}}_{mj} \rangle\}} \quad (10)$$

or the intersubject variance divided by the average intra subject variance.

The set of coefficients comprising a HRCV represents each segment of HR data as a point, described by a vector of p dimensions. Each cepstral vector for each subject is represented by a point in multidimensional parameter space. The vector set of S points forms a cluster with a centroid, described by (5), whose size is represented by the intrasubject variance and described by (8). Liftering the data for a population optimizes the ability to separate these clusters, and thereby identify individuals by their HRCVs. The optimal identification vector for a given subject would have a minimal intrasubject cluster size and maximal distance from all other subjects in the population. Hence, coefficients having the smallest average intrasubject variance and the greatest variance of intersubject averages should be weighted the highest, and vice versa, according to (10).

3 Experimental methods

3.1 Data collection and analysis

Subject data were obtained from the American Heart Association (AHA) database, consisting of annotated cardiograms from 80 cardiac patients. The criterion for

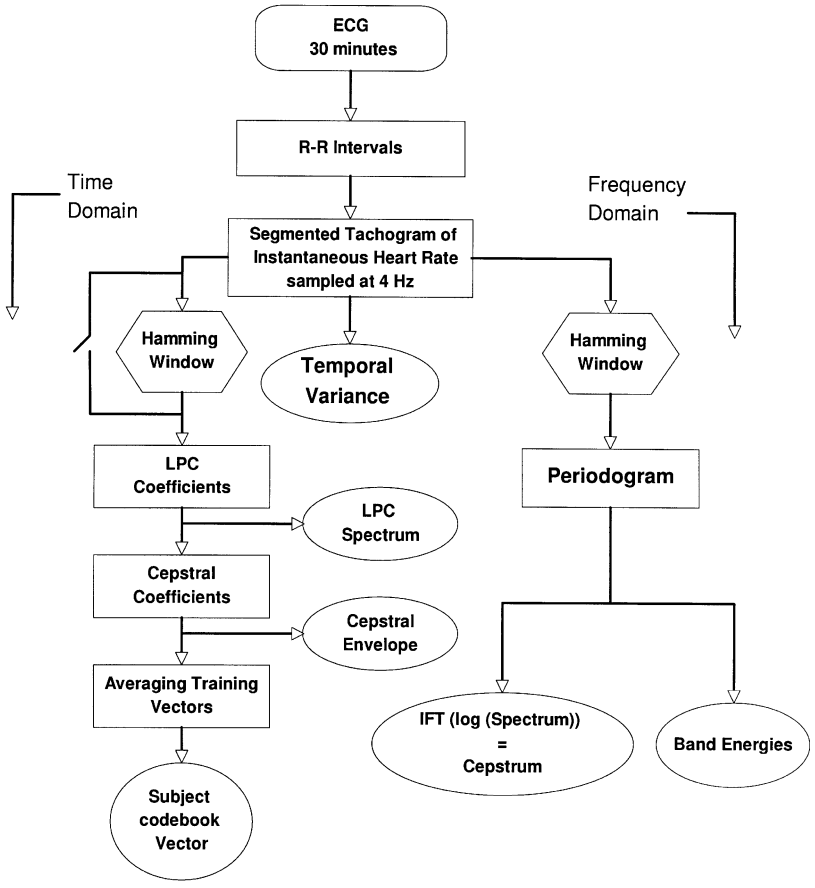


Fig. 1. Flow chart of data analysis

entry into the study was the presence of at least 25 min of stationary sinus rhythm, defined as a stable baseline with no observable trends or transients, and with fewer than 5 premature beats. A total of 12 subjects from the AHA records met this criterion. Beat-to-beat interval tachograms were provided by the AHA annotation files.

A flow chart of the data analysis procedures is depicted in Fig. 1. Initial records consisted of approximately 30–35 min of beat-to-beat intervals, representing a tachogram. Each record was examined for artifacts (missed or extra beats), and standard criteria and methods for correction were applied where necessary (Craelius et al. 1987; Bekheit et al. 1990). Records with excessive artifacts (>1 per 5 min) or non-correctable artifacts were rejected. Tachograms were converted to instantaneous heart rate (IHR), sampled at 4 Hz using the method of Berger et al. (1986).

In the time domain (Fig. 1, left side), the LPC coefficients were calculated using (1). The Hamming window was applied in order to correlate with time-domain with frequency-domain indices, but was omitted for subject recognition (see Sect. 3.2 and 3.3). The LPC coefficients generated the LPC spectrum and the cepstral coefficients were calculated from (2). For Sect. 4.1, the cepstral envelope spectrum was generated using (3). To obtain codebook vectors for each subject, HRCVs were determined on each of 11 adjacent IHR segments, representing 640 samples each. Segment size was thus 160 s (640 samples/(4 samples/s)), which is within the sample size recommended for spectral resolution of the three main spectral HRV indices (see below and Task Force 1996). The subject's codebook HRCV was computed as the ensemble average of the set of 11 segmental HRCVs, or the subject's vector set, using (5).

Computation of HRCVs was done in 'C' and run on a Pentium 90 MHz computer. The entire processing time for 30 min of IHR data was less than 1 s. The standard HRCV was represented by 12 coefficients, but the range from 8 to 24 was tested (see Sect. 4.1). For illustration purposes, the cepstrum was calculated from the log spectrum using MathCad (Mathsoft, Cambridge, Ma.). Two separate tests were applied to the HRCV, as described below, and in corresponding sections of results. MathCad and custom 'C' programs were used where noted.

3.2 Individual recognition

The accuracy of HRCVs as individual descriptors was estimated using within-group discrimination analysis, as depicted in Fig. 2. Weight vectors (lifters) were calculated from subject vector sets by (6) through (10). The lifters were normalized to a maximum of 1, and used in the

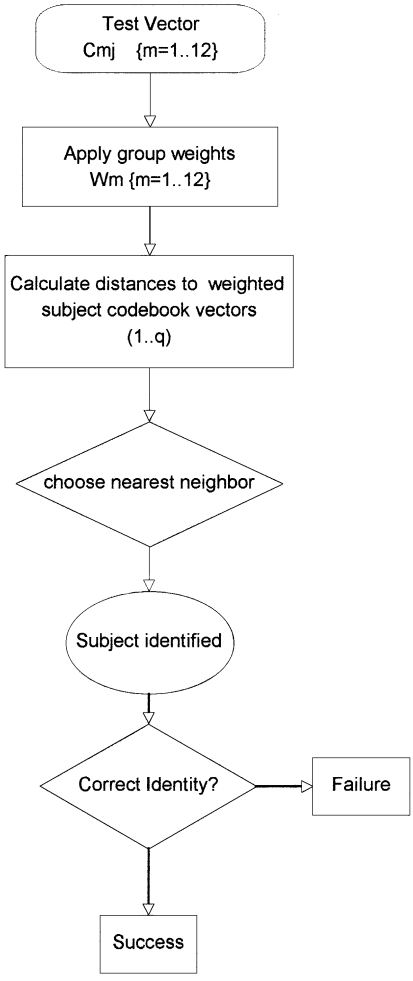


Fig. 2. Flow chart of subject identification

calculation of cepstral distance measures (4). Cepstral distances were then used to calculate a table of F-ratios using (O'Shaughnessy 1986):

$$F = \frac{\text{intersubject codebook } d_{\text{wcep}}}{\text{average intrasubject } d_{\text{wcep}}} \tag{11}$$

The F-ratio measures the cepstral distance for each subject in the group to every other subject in the group. For example, when comparing two subjects, A and B, subject A's codebook vector is 'F' times the average weighted cepstral distance of subject B's vector set, with respect to B's codebook vector. A group of N subjects will produce $N \cdot (N-1)$ F-ratios. Our group of 12 subjects thus contained 132 comparisons. Another more direct measure of HRCV reliability is based on the success rate of the nearest neighbor (minimum distance) method for identification of subjects (see Fig. 2). For this measure, test HRCVs taken from the group vector sets were weighted and compared against the group codebook, consisting of each individual's codebook HRCV. By simply equating the HRCV with its nearest codebook vector, based on minimal weighted distance (4), the fraction of correct identifications was determined directly.

3.3 Correlation

In order to identify possible physiological correlates of HRCV coefficients, the correlation between raw HRCV coefficients and several time domain and spectral measures was calculated. For each corresponding segment of IHR data used to calculate a HRCV, a record was made of the corresponding mean, standard deviation, and several spectral-band parameters. The HRV indices computed were spectral powers in the low frequency (LF, 0.03–0.15 Hz) and high frequency (HF, 0.15–0.4 Hz) bands, LF/HF ratio, and total power (0.03–0.4 Hz). The RSA peak is within the HF band. Spectral analysis was performed using the standard FFT algorithm in 'C', with a Hamming window applied to each data segment. HRCV coefficients were computed under identical windowing conditions to optimize correlation results.

4 Results

4.1 Cepstral analysis of heart rate

A typical analysis is depicted in Fig. 3, where Fig. 3a shows the IHR tachogram consisting of 1000 samples obtained from AHA #1207. The predominant periodicity in the record corresponds to RSA, occurring at about 13 per minute. The power spectrum of the segment, shown in Fig. 3b, has a broad peak near the fundamental RSA frequency between 0.20 and 0.25 Hz. Two other peaks occur in the 0.60–0.64 Hz band and 0.82–0.88 Hz band, that contain power within the second and third harmonics of the fundamental. The standard HRV indices are shown at *right*. The log power spectrum (Fig. 3c, solid line) shows the harmonic pattern more clearly. A pattern of spectral oscillation suggesting a fundamental frequency of about 0.22 Hz is apparent.

The LPC-derived spectral representation using 24 coefficients (Fig. 3c, dashed line) smoothly envelopes the spectrum. Since a pth order AR spectral estimate will always have p or fewer peaks and troughs, the degree of smoothing relates inversely to p, which need only be chosen large enough to adequately resolve the nearest adjacent peaks bearing significance for the results without performing needless computations. This can usually be accomplished by a small number of LPC coefficients, since it can be shown that the AR spectral model adequately represents the peaks (but not the valleys) of a periodogram (Kay and Marple 1981).

Figure 3d shows the LPC-derived spectra generated from 12 and 24 coefficients. Figure 3e similarly shows spectra calculated from cepstral coefficients (3) using 12 and 24 coefficients. The 12 cepstral coefficients for the data set are shown. Note that the 24-coefficient representation for both the LPC and cepstral vectors produces greater spectral resolution, as expected.

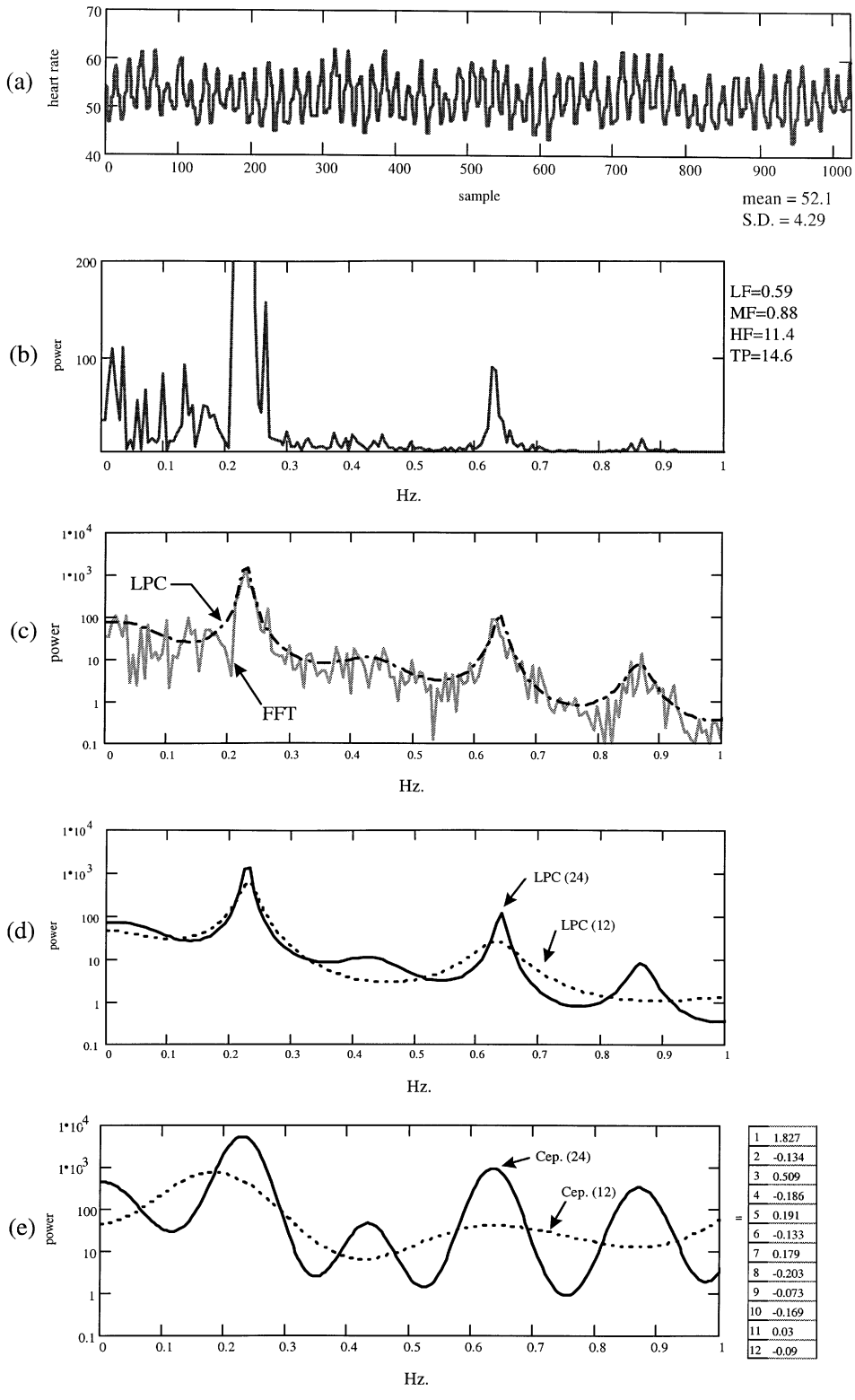
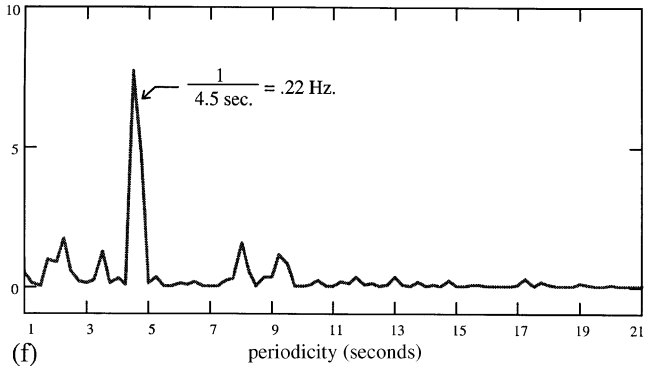


Fig. 3a–f. Spectral and cepstral analysis of heart rate (HR). **a** Instantaneous HR. 1024 samples are displayed, representing 4.2 min of continuous HR. **b** HR power spectrum of signal in **a**. Respiratory sinus arrhythmia (RSA) peak at 0.22 Hz is off-scale, to show details. Shown at *right* are spectral band powers in BPM^2. **c** Log power spectrum of **a**, with superimposed linear predictive coding (LPC) spectrum envelope (*dashed line*) estimated from 24 LPC coefficients (*FFT*, fast fourier transform). **d** LPC spectra based on 24 coefficients (*solid*, identical to **c**) and 12 coefficients (*dotted*). **e** Cepstral envelope estimates using 24 (*solid line*) and 12 (*dotted*) coefficients. **f** Cepstrum of **a**. *Abscissa* spans periods from 1 to 21, corresponding to 1–0.05 Hz. Peak at 4.5 s represents RSA at 0.22 Hz, equivalent to cardiac pitch



(**Fig. 3a–f.** Continuation)

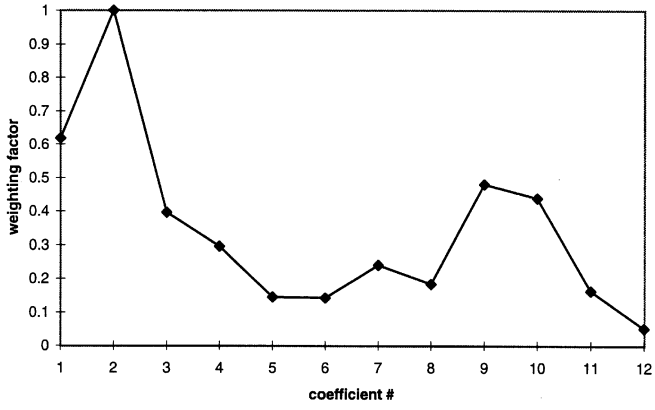


Fig. 4. Cepstral weighting vector. The weight of each cepstral coefficient (*abscissa*) was calculated using (10) and is shown on the *ordinate*. The weights represent the intersubject variance divided by the average intrasubject variance for the 12 selected patients in the AHA database. The weights were used in the calculation of cepstral distance to optimize recognition performance within the group [see (4)]

Finally, the cepstrum of the tachogram, in the frequency domain, is shown in Fig. 3f. The period scale of the cepstrum corresponds to a frequency span from 1 Hz on the left to 1/21, or 0.05 Hz, on the right. The prominent peak occurring at 4.5 s corresponds to the fundamental of 0.22 Hz, seen in Fig. 3b–e. The cepstrum thus emphasizes the fundamental, which in this case corresponds to RSA.

4.2 Individual recognition

Figure 4 shows the empirically determined weights for the subject group. Weights are high for coefficients having a high intersubject variance coupled with low intrasubject variance, and viceversa, according to (10). It can be seen that coefficient 2 has the maximal weight, followed closely by coefficient 1. In contrast, coefficients 5, 6, and 12 contribute minimal weight to the recognition process as defined by (4). These group-specific weights are used to lifter the HRCVs, in order to optimize the reliability of cepstral distance measures for identification.

The overall reliability of HRCV as a subject identifier is measured by the F-ratio (11), with unity representing no discriminability, and higher values indicating increasing discriminability. The average F-ratio for the group was 53 ± 61 and differed significantly from 1 ($P < 0.0001$). The actual success rates in identifying individuals by comparing their HRCVs against the relevant population codebook (see Sect. 3.2) was 123/132 or 93% for the subject population. The majority of the failures were due to one subject, whose variance was relatively high. Lower success rates resulted when the signal was represented by fewer than 12 vector coefficients (not shown).

4.3 Spectral-cepstral correlation analysis

Correlation between the mean of each standard HRV index and the individual HRCV coefficients showed that coefficient #1 correlated negatively with both standard deviation and total power of heart rate ($r = -0.69$ and -0.64, respectively; $P < 0.0001$). No other HRCV coefficient correlated significantly with standard HRV indices. Correlation values for LF, HF, and LF/HF were all $r < |0.6|$. This general lack of correlation, coupled with the ability of cepstral coefficients to reproduce the spectrum (Fig. 3d), suggests the novelty of HRCV as an HRV index.

5 Discussion

We tested the ability of multidimensional cepstral vectors (HRCVs) to classify individual HR patterns. Within a limited population we have shown that a 12-dimensional HRCV reliably characterizes HR and can identify individuals with an accuracy of 93%. This reliability compares favorably with that reported for speaker recognition (Atal 1976). Identification was achieved by minimizing the weighted distances between vector set and codebook HRCVs. Thus, the HRCV may represent a homomorphic filter for HR signatures, although a more extensive test is necessary to establish its long-term persistence. Most subjects reproduced highly stable HRCVs during 30-minute recording intervals.

Standard HRV indices of the subjects studied do not correlate strongly with HRCV, with the strongest found between coefficient 1 and HR standard deviation ($r = -0.69$; $P < 0.0001$). HRCVs can, nevertheless, reasonably reproduce the major spectral features, with as few as 24 coefficients (Fig. 3e). With 12 coefficients, HRCV loses spectral peak resolution, but retains sufficient information to discriminate among individuals. HRCV thus represents the entire HR pattern, with an accuracy dependent on the number of coefficients. Its recognition accuracy might improve with higher-dimensional vectors, at the expense of computation.

The sensitivity of the cepstrum to harmonic oscillations in speech is a major key to its ability to extract individual voice prints, that are largely based on pitch (Soong and Rosenberg 1988; McAulay and Quatieri

1992). Harmonic variations in HR, when present, are likewise emphasized by the cepstrum, as shown in Fig. 3b–e. In this subject, RSA behaves harmonically, and therefore appears prominently in the cepstrum, as the HR equivalent of pitch. Since RSA is partially generated by mechanical breathing vibrations, breathing harmonics in the cardiovascular system are not unexpected, and have been previously described (Craelius et al. 1986).

The HRCV, in select physiological states, may represent a 'heart-print', analogous to dermatological and voice patterns. Further refinements of HRCV analysis may enable individual as well as group classification according to identity as well as physiological status.

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